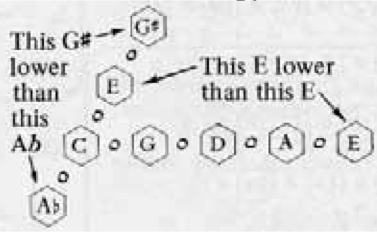
Tuning Systems for 12-note Keyboard Instruments

by Mark Lindley

In the 'panorama' of historical tunings shown on the following pages, the hexagon diagrams represent spirals in which F comes after Bfo in the horizontal sequence of 5ths. likewise A after D and C sharp after F sharp. The numbers in" the diagrams indicate how much each interval is tempered. A positive number means that the 4th, major 3rd or major 6th in question is tempered larger than pure; a negative number means the opposite (e.g. a 4th tempered smaller or 5th larger than pure).

The unit of measure is a tiny intervallic quantity known as the schisma, which approximates 1/100 of a whole tone and equals almost exactly 1/11 of the syntonic comma, 1/12 of the pythagorean comma, and 1/21 of the lesser diesis. These are, it may be recalled, three of the most notable 'wrinkles' among *pure* concords:

1. the syntonic comma (5ths vs 3rds): 11 schismas
2. the pythagorean comma (circle of 5ths): 12 schismas
3. the lesser diesis (major 3rds vs 8ve): 21 schismas



From these diagrams one can readily estimate the rate of beating in any consonant interval in the various tunings shown (assuming that the timbre is not severely inharmonic). First, find the pitch level of the lowest unison among the harmonic partial tones of the interval:

in a 5th, the second partial of the upper note makes a unison with the third partial of the lower note:

in a 4th, the third partial of the upper note makes a unison with the fourth partial of the lower note:

In a major 10th, the second partial of the upper note makes a unison with the fifth partial of the lower note:

in a major 3rd, the second partial of the upper note makes a unison with the fifth partial of the lower note:

in a major 6th, the third partial of the upper note makes a unison with the fifth partial of the lower note:

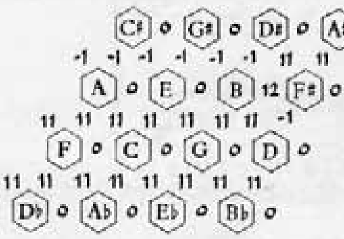
in a minor 3rd, the fifth partial of the upper note makes a unison with the sixth partial of the lower note:

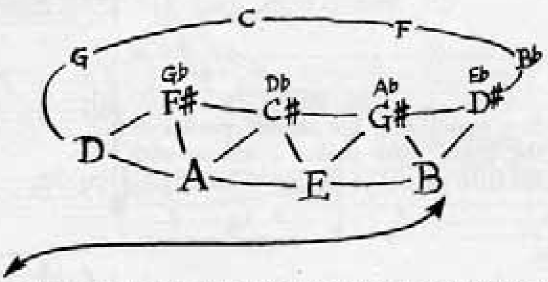
If the unison is at a 880, then the number of beats per second will be the same as the number of schismas. If the unison occurs at a higher or lower pitch level, the beats will be accordingly faster or slower. For example, if it occurs an octave below A 880, the beat rate will be half the number of schismas (because the frequency ratio for an octave is 2:1; if it occurs a 5th above A 880, the beat rate will be half again the number of schismas (because the ratio for a 5th is 3:2); and so on.

A Panorama of Historical Tunings

Untempered tunings

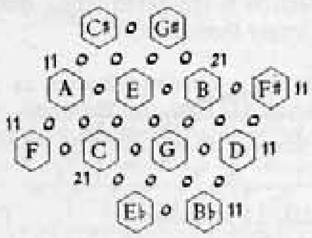
Pythagorean: comprising a chain of eleven pure fifths and one wolf fifth; hence 17 major thirds and 6ths are a comma larger then pure, but there are also 7 nearly pure thirds, e.g. an F# x B disposition (as recommended by a dozen 15th-century theorists:





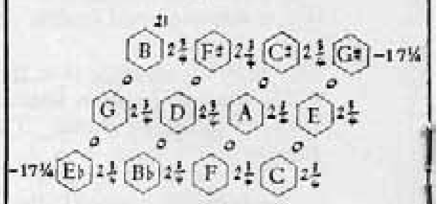
nearly pure thirds and sixths (perhaps unbeknownst to the tuner), specially employed in the liturgical pieces in the Faenza Codex (early 1400's) and in certain pieces (Nos. 19, 30, 31, 128, 141, 153–55, 180, 242) of the Buxheim Organ Book.

'Just intonation', avoided on normal keyboard instruments because of its three wolf fifths, e.g.:

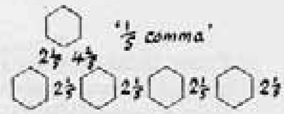


According to Ramos (1482) the wolf fifth was between C sharp and A flat. ¼-comma meantone was described by Zarlino (1571), Salinas, Praetorius, Mersenne et al.

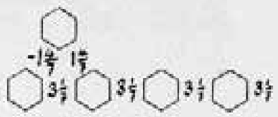
Meantone: with a large wolf fifth and a chain of eleven fifths tempered small by 2 – 3 schismas depending on whether the major thirds are to be pure:



or tempered slightly large (Lanfranco 1533; Verheyen c.1600; Cerone 1613; L. Rossi 1666; Sauveur 1701; G. Silbermann), e.g.:



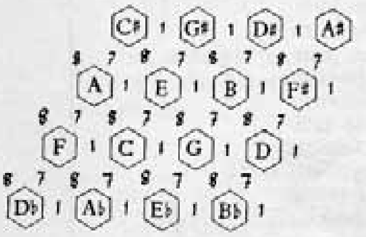
or perhaps slightly small, to the advantage of the major sixths, e.g. in Zarlino's 2/7 – comma scheme (1558):



Temperaments

irregular

equal temperament: with no wolf fifth; used on lutes from the sixteenth century and gradually adopted on keyboard instruments from the late seventeenth century (? Frescobaldi, 'Cento Partite sopra Passacagli'; Froberger; Rameau 1737; C.P.E. Bach; J.N. Ritter, German Organ Maker; Marpurg; Hummel; Cavaillé-Coll):



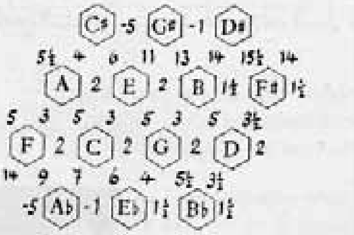
Syntonic comma = 11 schismas

Pythagorean comma = 12 schismas

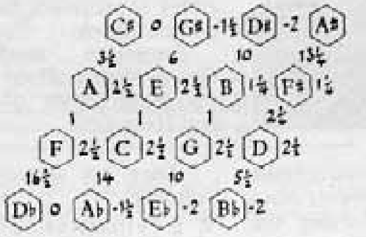
lesser diesis = 21 schismas

with a wolf fifth departing only very slightly from the absolute regularity of a strict meantone (Aron 1523; Schneegass 1590; Cima 1606; Milliet de Chales 1674; and perhaps most harpsichord tuners from the late 15th to the 17th century)

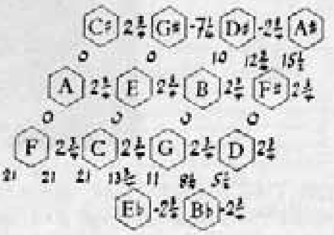
with something of a wolf fifth, e.g. Schlick 1511:



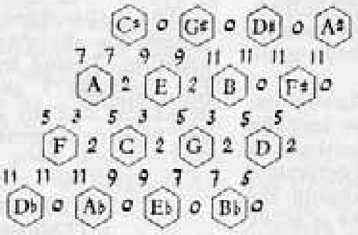
without any wolf fifth: French types from the late 17th century (L. Couperin, F sharp minor pavane; D'Anglebert) until c.1800 (temperament ordinaire) e.g.:



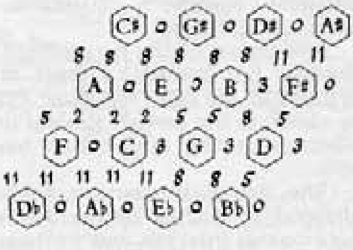
Mersenne (1635) via an inadvertent ambiguity concerning the fifths E flat–B flat–F (cf. L. Couperin, G minor passacaglia, etc.):



Vallotti:



Werckmeister 1680s (*gute Temperatur*; cf. Sorge 1748):



— and crude approximations of equal temperament.