

# The Mathematical Architecture of Bach's Goldberg Variations

by **Herbert Anton Kellner**

*Dedicated to Graf Arnold and Grafın Wilhelmine von Keyserling*

After having published in 1739 his *Drifter Theil der Clavier Übung*, Bach proceeded to finish about 1742 his last *Clavier Übung*. On the title page of the original print he discontinued the previous successive numbering: *Clavier Übung bestehend aus einer ARIA mit verschiedenen Veraenderungen vors Clavicimbel mit 2 Manualen*. This monumental harpsichord work is historically connected with Hermann Carl, Count von Keyserlingk (1696-1764), an admirer of Bach and enthusiastic lover of his art. Going even beyond that, it was due to his patron's dedicated intervention in his favour that Bach was ultimately nominated to the title 'Court Composer to the King of Poland and the Elector of Saxony', Friedrich August II. Count von Keyserlingk's harpsichordist Johann Gottlieb Goldberg was an extremely gifted virtuoso, congenial to the music he was first to perform: the Aria with its different variations, written by Bach *Denen Liebhabern zur*

*Gemiiths Ergetzung*, as stated on the title-page, 'To the amateurs for the enjoyment of the mind'. And plenty of time will indeed gently and agreeably flow by when listening to this magnificent music, from which Count von Keyserlingk, when tormented by sleepless nights, was cheered up and found consolation, asking his friend Goldberg, 'Do now play from *my* variations<sup>2</sup>'. This music is, in fact, the most extended single composition for the harpsichord Bach ever wrote. Despite that - in terms of performing-time or the number of records - this masterpiece is nevertheless structured in a surprising and almost incredible way. Not only by its two main sections and by the numbering of the variations, but there is also an internal *exordium*, the overture No. 16; in line with the rhetorics dating from Antiquity and resumed in the Baroque epoch<sup>1</sup>. There is even a sophisticated structuring in the count of bars across the oeuvre, connected

with Bach's esoteric and yet highly rational numerological mysticism, rooted in the cabbalistic numbering of letters.

In my two previous English Harpsichord Magazine articles<sup>3</sup> I have demonstrated that there exist some relations between the counting of bars across *Das wohltemperirte Clavier* and a set of numbers belonging to Bach's musical temperament. Such observations prove an unexpected unifying element between the keyboard temperament and the composition with which it is associated. In the numbering of bars through *Das wohltemperirte Clavier*, not only the initial and concluding bars of preludes and fugues can be taken into account, such as the first bar 577 of the Trinity E-flat major prelude, or 1913 and 1931, opening and winding up the prelude of the B-major tempering-key; but also intermediate bars. For example, counting through, bar 129 = 3 x 43 (Trinity x CREDO) sounds the C three - and even fourfold - this is bar 29 of the C-minor fugue, 29 being equivalent to Soli Deo Gloria, SDG, or, the composer's monogram JSB. Before I had discussed this peculiar passage in EHM II, it was already noted by Beckh - although he takes an approach towards the character of tonalities which is somewhat outdated by now<sup>4</sup>. Even the total of bars comprised within the WTC I was apparently controlled by the very mathematically minded composer. Their total number 2135 factorizes 2135=35x61. Already its C-major prelude has 35 bars, formed by the digits 3 and 5, third and fifth in figured bass. Or, 35=5 x 7, reflecting the 5 well-tempered and 7 perfect fifths of the tuning-system. The prime 61 may be related - typically numerologically - to the beat-ratio 6:1 of tempering the B-major triad. The total aggregate 21-35 by juxtaposition, may be looked at, somewhat artificially, as 21=3x7 together with 35=5x7, whilst 37 = J. CHR., the concatenation of the factors 3 and 7. So far we have considered the 2135 hearing -bars, and because of the repetition-form AABB of the B-minor prelude No. 24, there will be 47 bars less of 'composition-bars', amounting to 2135 - 47=2088. As this decomposes 2088=72x29, surprisingly, Bach's monogram JSB =29 reappears via this alternate mode of counting.

Along similar lines, as with the bars, the conclusion is inevitable that Bach has also controlled the number of keystrokes, at least in the instance of some pieces. Thus, the C-major prelude sounds 551=19x29 keystrokes: the tuning-steps times JSB. Or, the B-major prelude is formed by 417 keystrokes to reflect the numbers of 4 well-tempered fifths, the 1 tempering-fifth B-f sharp and the 7

perfect fifths, whilst 417=3x139, Trinity and 19 centred numerologically upon 3, i.e. 1-3-9.

Compared to the Well tempered Clavier, in the structure of the Goldberg Variations much greater regularity prevails. Apart from a few exceptions, all pieces measure 64 bars, counting, of course, the repetitions as requested by the composer. Therefore nothing comparable to the numbering within the WTC is to be expected. If at all, peculiar features could presumably only stem from the few irregularities, i.e. those pieces which deviate from the standard length of 64 hearing-bars. In fact, the variations No. 3, 9, 21 and 30 have only 32 bars, see e.g. Geiringer<sup>5</sup>. But the most striking exception is No. 16, the opening variation of the second part. Starting with a French ouverture, it has 95 bars, an odd number. Bach assured this count, by assigning one bar less to the repetition. Here, at the midpoint, 95 has the sum of digits 9+5 = 14, BACH putting himself into the centre of his composition, and it factorizes 95=5x19 with the number of the tuning-steps.

Let us now scrutinize Part I in more detail; it comprises the Aria and the first 15 variations. The second variation, after 2x64 = 128 bars (128=7 octaves by frequency), starts at 129=3x43 (CREDO=3 + 17+5+4 + 14=43). The onset of variation 3, canone alFUnisuono, is 3 x 64 +1 =193, a nice numerological triptych of the *uni-trinitas* 13 pivoted upon 9=3+3+3, *trias trinitatis per additionem*. This canon is also the first variation comprising only 32 bars. It thus closes at bar 224=2 x 112 (CHRISTUS = 112, see Hahn<sup>6</sup>). Differently, 224 = 16 X14, where 16 may be related to the beat-ratio 6:1 of the tempering, just as its crab 61 - which certainly can be used more significantly, as it is a prime. By his artifices, as applied up to this point, Bach ensures that the 5th variation starts with something like an apotheosis of the number 5, at bar 289 = 172=(5+7+5)z, having 2+8+9 = 19 as its sum of digits.

The first *numerus perfectus* is 6, made up by the sum of its divisors as follows: 6=1+2+3; such games were even even described in Johann Gottfried Walther's *Musicalisches Lexikon* of 1732<sup>7</sup> and by his teacher Werckmeister in *Musicalische Temperatur*<sup>8</sup>, 1691, chapter VI. Bach's 6th variation sets off with the prime triptych 353, the well-tempered fifths framed by the Trinity. It ends regularly after 64 bars at 416, at this stage rather arbitrarily decomposed into 41 for J. S. Bach and 6 again, *numerus perfectus*. But this also factorizes via 13 in a tri-unitarian way: 416=2<sup>5</sup> X13.

Much more significant is the initialization of the

next variation No. 7 - and undoubtedly a 'driving' aspect for numerological structuring, i.e.  $416 + 1 = 417!$  This is the well-known juxtaposition from the numbers of 4 well-tempered, 1 tempering fifth, and the 7 perfect fifths, equal to the count of keystrokes within the B-major prelude of WTC I, the *tempering-key*. To recall the components,  $417=3 \times 139$ , Trinity and 19, pivoted numerologically upon 3, to form 1-3-9. Variation 9, Canone alia Terza, ends at bar  $576=24 \times 24$ , with a squaring-apotheosis of 24, the number of accessible keys in well-tempering; see also EHM I, p. 33 on the C-sharp major prelude of WTC II. In order to assure this result, Bach had to allot once more the exceptional length of 32 bars to a piece. But a further reason becomes apparent: the next variation, No. 10, starts at  $576 + 1=577$  which is the prime characterizing the closure of the cycle in Bach's tuning: 5 well-tempered plus 7 perfect fifths close 7 octaves. The number 10, *unitas figurata*, Werckmeister generated from summing the Pythagorean *Tetraktys*  $1+2+3+4=10$ , see *Hypomne-mata MusiccP*, 1697, p. 2. In EHM 11 have pointed out that the Trinity E-flat major prelude of WTC I just starts with 577 as its first bar.

Variation 11, with the number of the crucial well-tempering step of the sequence<sup>10</sup>, begins at bar 641,  $6=\text{numerus perfectus}$  and  $41=J. S. Bach$ ; corresponding symmetrically to the formerly used 416. Advancing further by the standard length of 64 bars leads to variation No. 12 beginning at 705; Canone alia quarta. The digits are the numbers of 7 perfect and 5 well-tempered fifths. 705 may be decomposed  $5 \times 141$ , and this numerologically partitioned into 14, respectively 41. The prime-factorization yields  $705=3 \times 5 \times 47$ ; 3 for the third and 5 the fifth in figured bass, and  $47=DEUS$  ( $4+5+20+18$ ), according to Hahns. In WTC I the crab of 705 was employed by Bach: the prelude in d-minor, No. 6, *numerus perfectus*, starts with just this crab:  $507=3 \times 169=3 \times 13 \times 13$ , an extraordinary representation of 5 and 7 by tri-unitarian elements, or by the Trinity and 19, centered upon 6, as 1-6-9. Proceeding further, and regularly, by 64 bars, leads to remarkable factorizations again at variation 15, Canone alia Quinta, starting at bar 897. The sum of digits  $8+9+7=24$ , the accessible tonalities, and with 23, the number of the tempering-key B-major in the WTC, the tri-unitarian factorization reads  $897=3 \times 13 \times 23$ . Variation 15 closes in g-minor with bar 960 the First Part of the Goldberg Variations ( $960=2 \times 10 \times 48$ ;  $48=INRI$ ). The entry into this bar 960 is crowned by a suspension on  $c^3$ , presenting the centre of tonality.

The final bar's key is the minor one complementary to G-major; only these two keys Bach uses for the pieces of the Goldberg Variations. In another instance Bach has distinguished a midpoint by g-minor, namely the E-flat major prelude, WTC I of 70 bars ( $70=IESUS$ , as I have shown in EHM II). In that prelude, bar  $35=5 \times 7$ , g-minor is set, in which triad, the beat rates of major and minor third are at 5:7. Furthermore, still at the centre, variation 16, the overture, initiates the Second Part of the composition: at bar 961, its sum of digits  $9+6+1=16$ , affirming the ordinal number of that piece. But here at the midpoint, a pronounced tri-unitarian symbolism manifests itself; evident from the factorization  $961=31 \times 31$ . This number is to be looked at as one of the usual concatenations, i.e. Trinity and Unity, to yield 3-1.

At this instance an amusing numerological feature may be noted, concerning 13 and its crab 31. Their squares are 169 and 961, respectively. Thus, the squares as well as their roots have their digits in reversed order. This was discovered as early as 1767 by N. Brownell, see also Dickson<sup>11</sup>, Vol. I, p. 453. At this opportunity it may be remarked, that  $31 \times 13=403$ , these digits being numerologically related to  $43=CREDO$ , but this does not occur here.

A further distinction of the midpoint, dividing the composition according to the *unitas*, the baroque concept and symbol for the perfection, will now be established, derived from the overall number of bars. In the original printed edition there is after variation No. 30, the Quodlibet, an *Aria da Capo e Fine*, see the figure, from Neumann<sup>12</sup>, p. 271. I owe to Prof. Johann Sonnleitner, Zurich, the observation that, in line with baroque performing-practice, this *Da Capo* is to be executed *without* the repetitions AABB within the Aria. His remark, in fact, stimulated the entire present investigation. Thus, unless this abstention from repetition is observed, the overall count of bars will be wrong -to this fallacy J. J. Duparcq<sup>13</sup> fell victim in his otherwise excellent and stimulating treatise, 1975. If, on the contrary, the *Da Capo* is played correctly without repetitions, the Aria winding up the Goldberg Variations will only contribute 32 bars to the overall count. Prof. Sonnleitner also pointed out to me, provided the *Da Capo* of the Aria is taken as it should be, then the last bar of variation 15, being 960, will be the exact centre - barwise - of the composition. A striking confirmation of the *unitas* derived from the bars, in addition to the numbering by the items of the variations! Under such aspects, the 95 bars of variation 16 may be looked at  $95=48$

*Variatio 30. a 1 Clav. Quodlibet.*

*Aria da Capo e  
Fine*

+47. Here at the centre, INRI+DEUS=95 - an *additive* partition. Generally, such partitions are to be handled with great caution: they are much less significant than factorizations, e.g. 129 = 3 x CREDO. But I presume this aggregate 47/48 is the cause for the suspension on  $c^3$  in these bars in the treble of the 6-part Ricercar of *Musicalisches Opfer*. This particular passage is discussed and explained in the brilliant and revolutionary JAMS-article<sup>1</sup> by Ursula Kirkendale, p. 117. The surrepetitious mode of the tenor's unison-entry on  $g^\circ$ , sounded simultaneously by the bass, may be interpreted as a manifestation of the *unitas*, in bar 48 of the Ricercar. In analogy to this crowning  $c^3$  - suspension within the *Musicalisches Opfer*, a similar suspension on  $c^3$  led to the central bar 960 of the Goldberg Variations, bringing about the presence of the tonal centre C, as already remarked for variation 15.

In order to further emphasize the role of the Goldberg Variations' centre, and after having brought forward an additive partition INRI and DEUS via 95, it appears useful to delve into the structure and music itself of variation No. 16. Only two bars will be scrutinized and counting composition-bars without repetitions, these will be bars 9=3 +3 +3 of the French overture and bar 23, in the second part of this variation. The number 23 belongs to the B-major items, of the *tempering-key*, within the WTC.

Both annotations, for bars 9 and 23, start off from the fact that the *tempering fifth* B-f<sup>o</sup> sharp has its nearly co-incident harmonics at f<sup>1</sup> sharp: this is the third harmonic of B and at the same time, the second partial of f<sup>o</sup> sharp; at this pitch off<sup>1</sup> sharp the tempering fifth's beats will be perceived. Within the B-major triad its third B-d<sup>o</sup> sharp must beat at 6

times the fifth's rate; the common harmonics, where the beats occur, are at  $d^2$  sharp. This is the fifth partial of B, and simultaneously, the fourth partial of the third  $d^\circ$  sharp. Now, in bars 9 and 23, the constellations ( $f^1$  sharp,  $d^2$  sharp) occur within the musical texture, at the centre of these bars.



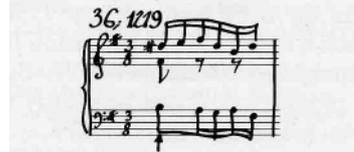
Bar 9, *trios trinitatis per additionem*, will be at the same time bar  $960+9=969$ , within the entire Goldberg Variations;  $9+6+9=24$ , the number of accessible keys. It also is a triptych, the first perfect number 6, centered within a frame of two digits  $9=3+3+3$ . But the factorization is extraordinary as well:  $969=3 \times 17 \times 19$ , or, Trinity times  $(5+7+5)$  times  $(7+5+7)$ , with the numbers of well-tempered and perfect fifths composing this way the factors.

The second occurrence of  $f^1$  sharp,  $d^2$  sharp at the centre of a bar, is in No. 23. Otherwise, this will be bar  $39=3 \times 13$  - tri-unitarian. Of the Goldberg Variations, this is bar  $960+39=999$ , factorizing at its sum of digits 27, by  $999=27 \times 37$ . One recognizes the trias trinitatis per multiplicationem  $3 \times 3 \times 3$  and  $37=J$ . CHR. At first sight, this nearly exaggerates the *trinitarian* aspect, but at the second passage, the repetition, the balance will be restored: now it is hearing-bar No. 1031, formed with the *unitas* being incorporated.

The most exciting numerology Bach has coded into Variatio 19 a 1 Clav., this is the number of tuning-steps. It starts at bar  $1184=2^5 \times 37$  - in the name of J. CHR. and by  $1+1+8+4=14$ : Bach. The partition 11-84 reflects the number 11 of the crucial tempering-step within the procedure<sup>10</sup> and  $84=6 \times 14$ , with 6, *primus numerus perfectus* of the beat-ratio and again 14, the composer! For literature on '84' see EHM I and II, for a historical source, see Hocker<sup>16</sup>. This variation Bach concludes at  $1247=29 \times 43$  - JSB and CREDO. Taken as a juxtaposition, it shows 12-47: the traditionally 'holy' number 12, also distinguished by Luther, see also Schmidt<sup>17</sup>, p. 32. There are 12 chromatic steps to the octave, and the remainder 47 is equivalent to DEUS.

This profound symbolism suggests looking into the music of variation 19, along the lines of what has been done for variation 16. Indeed, not many preparations are necessary here. In the B-major

triad, upon tempering, the third B- $d^\circ$  sharp must beat at 6 times the rate of the fifth B- $f^\circ$  sharp; their respective common harmonic partials are at  $d^2$  sharp and at  $f^1$  sharp. But just these notes are sounded after 19 composition-bars of variation 19, together with  $b^\circ$  of the triad. In terms of the hearing-bars of the musical execution, this bar with the beating partials of the tempering-triad, will be bar  $36=6 \times 6$  - the beat-ratio squared. For the Goldberg Variations, this is bar 1219, from  $1183 + 36$ . But 1219 shows the 12 chromatic steps and the 19 tuning-steps, and factorizes  $1219=23 \times 53$ ! And



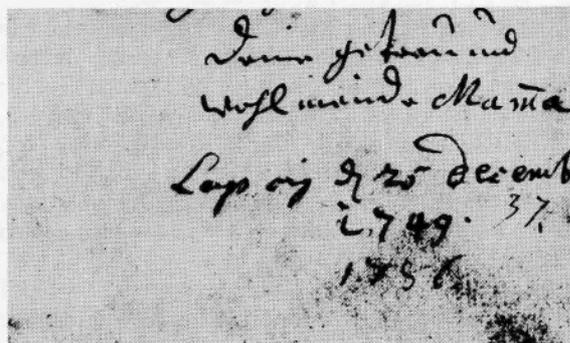
23 is B-major in the WTC and 53 fifth and third in figured bass. But upon repeating, at the second passage, this bar will become  $1235=5 \times 13 \times 19$ ; the well-tempered fifths, unity 1 and Trinity 3, with, unbelievably, again the 19 tuning-steps. 1235 may also be broken up into the juxtaposition 12-35, the steps to the octave and 3 with 5 of figured bass. Very skillful mathematics indeed, connected by Bach with the beating partials! The notable implications of variations 20,  $21=3 \times 7$  and  $27=3 \times 3 \times 3$ , the Canon at the 9th, may be left for the reader, by giving the following table. Variation Begin End

20	1248	1311
21	1312	1343
27	1664	1727

Concerning variation 29 - SDG, JSB, it ends at bar  $1855=5 \times 7 \times 53$ : the 5 well-tempered, the 7 perfect fifths and 53, via figured bass, fifth and third.

The concluding Aria starts at 1888, to be interpreted as 1-888: the *unitas* and  $888=24 \times 37$ . Here all 24 tonalities are symbolized and 37 signifies the monogram J. CHR. Resorting to such juxtapositions could seem suspicious, were it not that exactly this style was common in the Baroque epoch. See e.g. Harsdoerffer<sup>14</sup> for 1666 and 666, the beast from the apocalypse. Bach made use as well from such numerological mannerism in many instances which can be demonstrated by cross-references to EHM I and II. He terminates in his WTC I the g-minor prelude of 19 bars at  $1369=37 \times 37$ , or 1-369 with  $369=41 \times (3+3+3)$ . Or, ibidem, Fuga 19 in A major, starts at  $1577=19 \times 83$ , whilst in 1-577 we see the *unitas* and the prime belonging to the cycle of fifths. Let us now resume the concluding Aria of the

Goldberg Variations, for the first bar of which we had found 1888 with  $888=24 \times 37$ . Using the Greek system of the gnostics, as expounded in the voluminous book of Friesenhahn, 888 just means IESOUS, in contradistinction to the apocalypse's beast 666. Concerning 37 and its interpretation as J. CHR. it would be most desirable to have a helpful hint from Bach's time or even his surroundings. In fact, Bach's son Johann Christoph Friedrich was given a German Luther-Bible by Anna Magdalena Bachin, his *getreue und wohlmeinende Mama*. This bible's front cover is embossed with A.M.B. and the back with the year 1738. Into this present Bach's wife wrote a dedication for her son, dated, Leipzig 25th December 1749. Surprisingly, under the month, a '37' can be clearly seen as the figure shows, see Neumann 12, p. 279.



Finally, the total of the Goldberg Variation's bars will be checked from

26 items a      64 bars              1664

1 item a        95 bars              95

5 items a      32 bars              160

Total 32 items                      1919 bars

Thus  $1919 = 19 \times 101$ , reduplicates in juxtaposition the number 19 of tuning-steps. The remainder of the factorization, 101, alludes to the ratio of the *untas* numerologically.

It can be concluded that also in his Goldberg Variations, despite their quasi-regular structure, Bach has succeeded in underlining his crucial numbers 417 and 577 once more. In addition, he has introduced a wealth of further numerological and mathematical relations. The spiritual climax appears to be his personal CREDO in variation 19, by the factors of  $1247=JSB \times 43$ . Further highlights pointed out were related to the beating partials fi sharp and d<sup>2</sup> sharp which occur whilst well-tempering<sup>10</sup>. Bach used allusions to that in bars 999 and 1219. My curiosity was so aroused by these passages whilst playing them, such that I was prompted to look for a possible mathematical

background. Of course I had known that Bach rather frequently employs these two beating pitches within his music for exposed passages and pronounced effects. On the whole, Bach did not fail to utilize the most variegated manifestations of tri-uni-tarian implications for his *ars inveniendi*. From his keyboard temperament, the well-tempered triad of C-major may be looked at, in fact, as Bach's *nucleus mysticus musicae*<sup>TM</sup>

Within the Goldberg Variations, even the distribution of the canons entails a trinitarian allusion. From the Canone all'Unisuono, variation 3, they continue at variation 6 (a 1 Clav.), 9 ..., until Variatio 27, Canone alia Nona. Thus, 3, 6, 9, . . ., every third variation is a canon. An interesting relation between this kind of contrapuntal writing which Bach mastered with incredible superiority to produce delightful music, and the domain of tuning and temperament, must be mentioned here. It is based upon the fact that the monochord also used to be called *Canon Harmonious*, see under this term in the lexicon of J.G. Walther?, 1732.

For performing the Goldberg Variations, it must be stressed that the repetitions as requested by the composer are definitely not at the discretion of the player, there is no *ad libitum*. On the contrary, it is by far preferable to proceed exactly according to what Bach prescribes. And on records, there cannot possibly be an excuse for mutilating the superb music by not playing all the repetitions. In this respect, Mireille Lagace's recording deserves the highest praise (Cal. 1652 and 1663, cassette 4552). For a review, see EHM, Vol. 2/7, October 1980, page 171. Also, the interpretation by Blandine Verlet, faithfully respects the repetitions (PHI 6768074).

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