Tuning Systems for 12-note Keyboard Instruments
by Mark Lindley

In the 'panorama' of historical tunings shown on the following pages, the hexagon diagrams represent spirals in which F comes after B♭ in the horizontal sequence of 5ths, likewise A after D and C sharp after F sharp. The numbers in the diagrams indicate how much each interval is tempered. A positive number means that the 4th, major 3rd or major 6th in question is tempered larger than pure; a negative number means the opposite (e.g. a 4th tempered smaller or 5th larger than pure).

The unit of measure is a tiny intervallic quantity known as the schisma, which approximates 1/100 of a whole tone and equals almost exactly 1/11 of the syntonic comma, 1/12 of the pythagorean comma, and 1/21 of the lesser diesis. These are, it may be recalled, three of the most notable 'wrinkles' among pure concords:

1. the syntonic comma (5ths vs 3rds): 11 schismas
2. the pythagorean comma (circle of 5ths): 12 schismas
3. the lesser diesis (major 3rds vs 8ve): 21 schismas

From these diagrams one can readily estimate the rate of beating in any consonant interval in the various tunings shown (assuming that the timbre is not severely inharmonic). First, find the pitch level of the lowest unison among the harmonic partial tones of the interval:

If the unison is at a 880, then the number of beats per second will be the same as the number of schismas. If the unison occurs at a higher or lower pitch level, the beats will be accordingly faster or slower. For example, if it occurs an octave below A 880, the beat rate will be half the number of schismas (because the frequency ratio for an octave is 2:1; if it occurs a 5th above A 880, the beat rate will be half again the number of schismas (because the ratio for a 5th is 3:2); and so on.

© 1977 by Mark Lindley
A Panorama of Historical Tunings

Pythagorean: comprising a chain of eleven pure fifths and one wolf fifth; hence 17 major thirds and 6ths are a comma larger than pure, but there are also 7 nearly pure thirds, e.g. an F# x B disposition (as recommended by a dozen 15th-century theorists):

Just intonation', avoided on normal keyboard instruments because of its three wolf fifths, e.g.:

Meantone: with a large wolf fifth and a chain of eleven fifths tempered small by 2 - 3 schisms depending on whether the major thirds are to be pure:

According to Ramos (1482) the wolf fifth was between C sharp and A flat. 9/4-comma meantone was described by Zarlino (1571), Salinas, Praetorius, Merseine et al.

or perhaps slightly small, to the advantage of the major sixths, e.g. in Zarlino's 2/7 - comma scheme (1558):

© by Mark Lindley 1977
equal temperament: with no wolf fifth, used on lutes from the sixteenth century and gradually adopted on keyboard instruments from the late seventeenth century (Frescobaldi, Canto Partite sopra Passacaglia; Froberger, Rameau 1737; C.P.E. Bach; J.N. Ritter, German Organ Maker; Marpurg; Hummel; Cavaillé-Coll).

with a wolf fifth departing only very slightly from the absolute regularity of a strict meantone (Aron 1523; Schneeberger 1590; Cima 1606; Milliet de Chales 1674; and perhaps most harpsichord tuners from the late 15th to the 17th century)

with something of a wolf fifth, e.g. Schlick 1511:

without any wolf fifth: French types from the late 17th century (L. Couperin, F sharp minor pavane; D'Anglebert) until c.1800 (temperament ordinaire) e.g.:

Mersenne (1635) via an inadvertent ambiguity concerning the fifths E flat–B flat–F (cf. L. Couperin, G minor passacaglia, etc.):

Werckmeister 1680s (true Temperament; cf. Sorge 1748):

Syntonic comma
= 11 schisms

Pythagorean comma
= 12 schisms

lesser diesis
= 21 schisms

Vallotti:

and crude approximations of equal temperament.